NUMERICAL STUDY OF THERMAL INSTABILITY DURING HEATING OF CERAMICS

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This paper describes the results of a numerical study of the thermal instability that develops during heating of a ceramic whose coefficient of thermal conductivity diminishes with rising temperature.

It was previously shown theoretically [1] that thermal instability can arise in a system composed of a heater and a ceramic as a consequence of the dependence of the ceramic's coefficient of thermal conductivity on temperature (a diminishing or nonmonotone relationship). The number of stable and unstable steady-state regimes were determined within the framework of the steady-state approach, and the critical conditions for the transitions from one regime to another were established. Such nonsteady-state characteristics of thermal instability as the process stabilization time, induction period, dynamics of temperature field variation in the heater and ceramic, and so forth are of practical importance. The present paper is devoted to determination of these characteristics and numerical illustration of various thermal regimes during indirect electrical heating of a ceramic with a diminishing temperature function for its coefficient of thermal conductivity.

We will consider the following problem for this purpose. Suppose that we have a cylindrical heating element of radius r_0 (region 2, Fig. 1) enclosed in a layer of ceramic with thickness $d = r_1 - r_0$ (region 1, Fig. 1). An electric current is passed through the heater. The temperature drop along the length of the heater and in the ceramic layer is neglected, since it is considered to be small in comparison with the radial drop, because of the long ($\ell \gg r_0$) length. A constant temperature T_0 is maintained at the surface of the ceramic, or the heat rejection into the environment (calculated from Newton's law) is specified; we will assume the heat capacities of the heater and ceramic (C_2 and C_1 respectively) to be constant.

The formulation of the problem includes the unsteady-state heat conduction equations for the ceramic (index 1) and heater (index 2):

$$c_1\rho_1 \frac{\partial T_1}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_1(T_1)\, r\, \frac{\partial T_1}{\partial r}\right), \tag{1}$$

$$c_2\rho_2 \frac{\partial T_2}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_2(T_2)\, r\, \frac{\partial T_1}{\partial r}\right) + q_+(T_2), \tag{2}$$

with the boundary conditions:

$$r = r_1:\ T_1 = T_0 \left(\text{ or }\quad \lambda_1(T_1)\frac{\partial T_1}{\partial r}\Big|_{r=r_1} = -\alpha(T_1 - T_0)\right). \tag{3}$$

The contact at the heater-ceramic boundary is presumed to be ideal; we are therefore justified in specifying equality of the temperatures and heat fluxes:

$$r = r_0:\quad \begin{aligned} &T_1 = T_2,\\ &\lambda_1(T_1)\frac{\partial T_1}{\partial r} = \lambda_2(T_2)\frac{\partial T_2}{\partial r}. \end{aligned} \tag{4}$$

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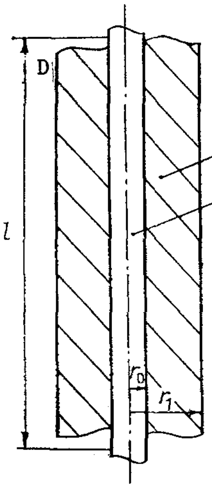

Fig. 1

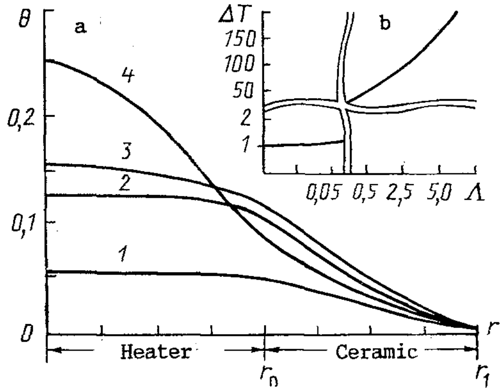

Fig. 2

Fig. 1. Diagram of system. 1) Ceramic; 2) heating element.

Fig. 2. a) Temperature distribution in system with different values for parameter Λ: 1) 0.004; 2) 0.048; 3) 0.48; 4) 4.83; b) maximum temperature drop in heater as function of parameter Λ. Here ΔT is in K and r is in mm.

The symmetry condition

$$\frac{\partial T_2}{\partial r} = 0 \tag{5}$$

is satisfied when r = 0. The initial conditions are:

$$t = 0:\ T_1 = T_{i1},\ T_2 = T_{i2}. \tag{6}$$

Passage of an electric current through the heater results in liberation of Joule heat in it, which is evaluated from the following formula for the constant-current regime [2]:

$$q_+(T_2) = \rho(T_2)\, j^2. \tag{7}$$

The function ρ(T) can be represented in the form [3]:

$$\rho(T_2) = \rho_0(1 + \gamma T_2), \tag{8}$$

for materials with metallic-type conduction. The heat source equation is written as follows in dimensionless form:

$$q_+(\Theta) = q_+(T_2)\, r_0^2/\lambda_{20}\Delta T. \tag{9}$$

The ceramic's coefficient of thermal conductivity is assumed to be a diminishing function of temperature [4]:

$$\lambda_1(T_1) = -aT_1 + b. \tag{10}$$

The dimensionless formulation of this problem is:

$$A\frac{\partial\Theta_1}{\partial\tau} = \frac{1}{x}\frac{\partial}{\partial x}\left(\Lambda_1(\Theta_1)\, x\, \frac{\partial\Theta_1}{\partial x}\right), \tag{11}$$

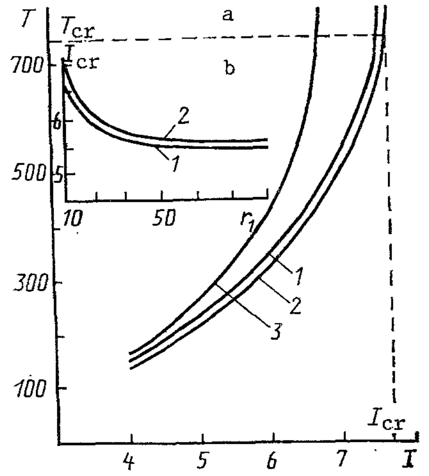

Fig. 3

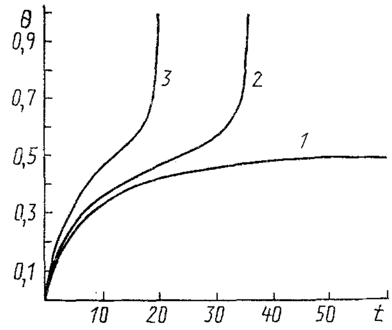

Fig. 4

Fig. 3. a) Steady state temperature as function of current under subcritical conditions: 1) steady state solution; 2) full formulation; 3) averaged formulation; b) critical current as function of ceramic layer radius with r_0 = 0.25 mm. Here I is in A and T is in °C.

Fig. 4. System heating curves for different currents: 1) I = 7.6; 2) 7.8; 3) 8.0 A. Here t is in sec.

$$\frac{\partial \Theta_2}{\partial \tau} = \frac{1}{x}\frac{\partial}{\partial x}\left(\Lambda_2(\Theta_2)\, x\, \frac{\partial \Theta_2}{\partial x}\right) + q_+(\Theta_2); \tag{12}$$

with the boundary conditions:

$$x = 0:\ \frac{\partial \Theta_2}{\partial x} = 0, \tag{13}$$

$$x = 1:\ \Theta_1 = \Theta_2,\ \Lambda_2(\Theta_2)\frac{\partial \Theta_2}{\partial x} = \Lambda_1(\Theta_1)\frac{\partial \Theta_1}{\partial x},$$

$$x = x_1:\ \Theta_1 = 0\ \left(\text{ or }\ \Lambda_1(\Theta_1)\frac{\partial \Theta_1}{\partial x} = -\mathrm{Bi}\,\Theta_1\right);$$

and the initial conditions:

$$\tau = 0:\ \Theta_1 = \Theta_{i1},\ \Theta_2 = \Theta_{i2}. \tag{14}$$

We employed the heat balance method [5] and a conservative balance scheme [6] to solve system of equations (11)-(14). The partitioning was uniform in region 2, while the space step in region 1 was consistent with the net in region 2. The finite difference equations were solved by the iterative trial-and-error method. The temperature regimes in the heater and ceramic were found by numerical solution for specific input data.

Figure 2 shows the temperature distribution in the heater and ceramic for different values of the parameter $\Lambda = \lambda_{10}/\lambda_{20}$, which characterizes the ratio of the coefficients of thermal conductivity for the heater and ceramic materials (Fig. 2a), and the maximum temperature drop in the heater as a function of the parameter Λ (Fig. 2b). The smaller the value of Λ, the more uniform was the temperature distribution in the heater. As our numerical calculations showed, the maximum temperature drop in the heater did not exceed 1 degree when the value of Λ was <0.05. It is best in this case to solve the simplified problem in which the heater temperature is averaged, introducing the average function $\bar{T}_2$:

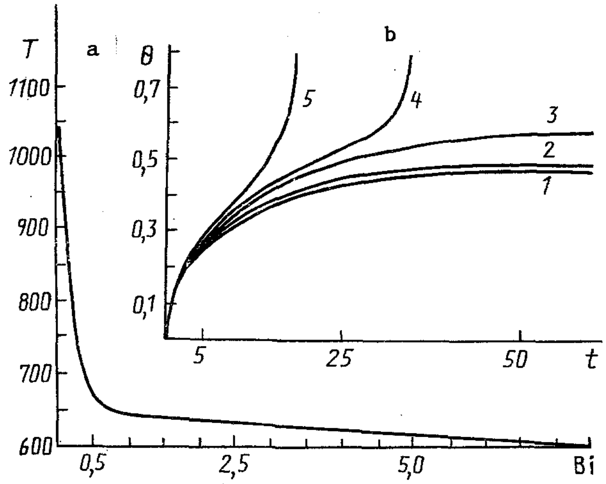

Fig. 5. a) Temperature at characteristic point as function of Bi, with I = 7.5 A, r_0 = 0.25 mm, and r_1 = 2.5 mm; b) system heating curves at different Bi: 1) Bi = ∞; 2) 5.0; 3) 0.5; 4) 0.25; 5) 0.

$$\overline{T}_2 = \frac{2}{r_0^2}\int_0^{r_0} T_2 r dr. \tag{15}$$

This formulation of the problem contains only one heat conduction equation for the ceramic:

$$c_1\rho_1 \frac{\partial T_1}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_1(T_1) r \frac{\partial T_1}{\partial r}\right). \tag{16}$$

The boundary conditions at the heater–ceramic boundary are written as:

$$c_2\rho_2 \frac{\partial \overline{T}_2}{\partial t} = \frac{2}{r_0}\lambda_1(T_1)\left.\frac{\partial T_1}{\partial r}\right|_{r=r_0} + q_+(\overline{T}_2). \tag{17}$$

This problem was also solved on a computer. As can be seen from Fig. 3, the results yielded by solution of the steady state problem in [1] (curve 1) and the full and averaged problems (curves 2 and 3 respectively) were in good agreement. Determination of the critical currents when heater geometric dimensions are small (in comparison with those of the ceramic) requires more computer time for the full than for the averaged problem, although the solution is more accurate in the first case. We therefore employed model 2 (the averaged problem) to determine the order of magnitude of the critical currents and then solved the full problem (model 1) near these values in order to find the specific parameters.

The computational results in Fig. 3a enabled us to establish that a stable steady-state regime was realized in the system when the current was below the critical level. The maximum steady-state temperature did not exceed 800°C in this case. A progressive heating regime (the so-called exacerbation regime [7]) appeared at supercritical currents, leading to system failure.

Figure 4 illustrates system dynamics (with the heater–ceramic boundary as the characteristic point) under subcritical (curve 1, current of 7.6 A) and supercritical (curve 2, with a current of 7.8 A) conditions. As can be seen, the transition from steady-state to essentially unsteady conditions took place discontinuously with only a small change in current. The current under supercritical conditions had a significant influence on the induction period for the exacerbated regime (curves 2 and 3). We will follow [8] in assuming the induction period to be the time required to attain the maximum heating rate.

The critical current was affected by the ceramic wall thickness and the conditions for heat transfer to the outer surface of the system. Figure 3b shows the influence of ceramic layer thickness on the critical current. As can be seen, the sharpened values for the critical current (unsteady formulation, curve 2) were somewhat higher than those yielded by solution of the steady-state problem (curve 1). It should be noted that, once the ceramic layer reached a certain thickness, making the layer thicker did not lead to any perceptible change in the critical current.

The computational results depicted in Fig. 5 enable us to consider the influence of external heat transfer conditions on system heating regime. It can be seen (Fig. 5a) that the influence of heat rejection to the outside on the thermal conditions in the ceramic was negligible as Bi was raised from 0.5 to ∞. This case is similar to that in which a constant temperature is specified at the surface. Variation of Bi over this range did not lead to any perceptible change in heating conditions (Fig. 5b, curves 1 and 2). Variation of Bi in the interval]0; 0.5[had a strong influence on the induction period and caused a qualitative change in heating regime, leading to a transition from subcritical to supercritical conditions (curves 3 for the first case and 4 and 5 for the second in Fig. 5b).

Analysis of the results described in the present paper indicates that the nonsteady-state characteristics and the parameter region corresponding to thermal instability have real values during electric heating of a ceramic and correspond to practical design calculations for heat insulation and operation of heating equipment.

SYMBOLS

r_0, r_1) radii of heating element and ceramic respectively; ℓ) heater length; T_0, T_1, T_2) temperatures at system outer surface, in ceramic, and in heater; T_*) characteristic temperature of heater; c_1, ρ_1, c_2, ρ_2) heat capacities and instability of ceramic and heater; λ_2, λ_1) coefficients of thermal conductivity for heater and ceramic materials; $\rho_1(T_2)$ heater resistivity; $q_+(T_2)$) heat evolution flux instability; $j = I^2/\pi^2 r_0^4$) current instability; I) heater current; t) time; $\Theta = (T-T_0)/(T_*-T_0)$) dimensionless temperature; $x = r/r_0$) dimensionless coordinate; $\tau=\frac{a_{20}}{r_0^2}t$) Fourier number; $Bi = r_1/\lambda_{10}$) Biot number; a_{10}, a_{20}) coefficients of thermal diffusivity for ceramic and heater; λ, A, D, D_1) parameters, defined as $\Lambda=\lambda_{10}/\lambda_{20}$, $A=a_{10}/a_{20}$, $D=\frac{T_0}{\Delta T}-\frac{b}{a\Delta T}$, $\Delta T=T_*-T_0$, $D_1=b_1/a_1\Delta T$, where a, b, a_1, and b_1 are empirical constants in relationships $\lambda_1(T_1)=-aT_1+b$, $\lambda_2(T_2)=a_1T_2+b_1$, $\lambda_{10}=a\Delta T$, $\lambda_{20}=a_1\Delta T$; $\Lambda_1(\Theta_1)$, $\Lambda_2(\Theta_2)$) dimensionless functions $\Lambda_1(\Theta_1)=-(\Theta_1+D)$, $\Lambda_2(\Theta_2)=\Theta_2+D_1$.

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